1) Suppose you have the following vectors/matrices with indicated dimensions:

$$A(=3\times 5);\quad B(=6\times 10);\quad C(=2\times 10);\quad D(=5\times 5);$$

$$x(=6\times 1);\quad y(=10\times 1);\quad z(=5\times 1).$$

What are the resulting dimensions of the following calculations:

$$AD,\ DA,\ BC,\ BC^T,\ x^TB,\ By,\ ADz,\ x^TBy?$$

If you cannot perform an operation, state why not. Also, if the result is a vector, state whether it is a row vector or column vector.

2) Given the vectors

$$x=\begin{pmatrix}1\\2\end{pmatrix},\quad y=\begin{pmatrix}3\\-1\end{pmatrix}$$

compute the following: $x-y,\ x+2y,\ x^T y,\ y^T x$.

3) Given the quantities

$$A=\begin{pmatrix}-3 & 1\\ 0 & 4\\ 1 & -2\end{pmatrix};\quad B=\begin{pmatrix}1 & 2 & 3\\ -1 & 2 & -1\end{pmatrix};\quad x=\begin{pmatrix}2\\1\end{pmatrix};\quad y=\begin{pmatrix}1\\-1\\-2\end{pmatrix}$$

compute the following: $A^T,\ Ax,\ A^Tx,\ y^TA,\ y^TB,\ AB,\ BA$. If a quantity cannot be computed, state the reason why.

4) For $A$ and $x$ as defined above, compute $\|x\|_1,\ \|x\|_2,\ \|x\|_\infty,\ \|A\|_1,\ \|A\|_\infty,\ \|A\|_F$.

5) Prove that

$$\|x\|_1=\sum_{i=1}^{n}|x_i|$$

is a norm (*i.e.,* show that this definition satisfies conditions i) - iv) in Section 4 of the Linear Algebra Handout).

6) Prove that

$$\|x\|_2^2=x^Tx.$$

7) Prove that

$$\lim_{p\to\infty}\|x\|_p=\|x\|_\infty.$$

8) (Extra Credit) Prove that

$$\|A\|_{\max}=\max_{\substack{i=1,\ldots,n\\ j=1,\ldots,m}}|a_{i,j}|$$

is not a norm (HINT: Find 2 matrices that violate condition v) in Section 4).