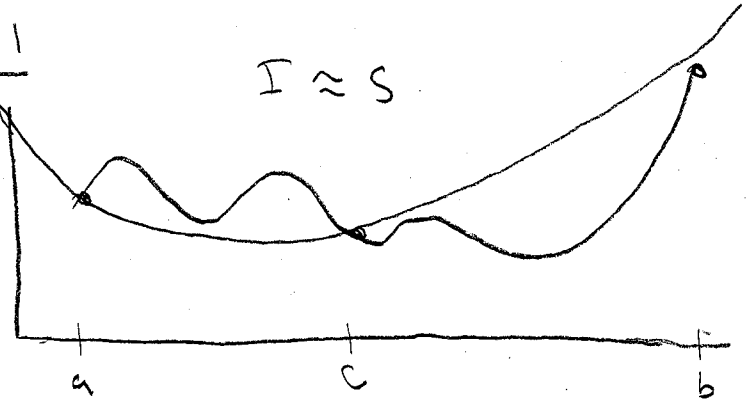
Adaptive Simpson's Rule

- Assumes $f(x)$ can be evaluated at any x
- cost is measured in number of times that $f(x)$ needs to be evaluated
  - we have been looking at simple functions
    $f(x) = e^{-x}\cos(x)$
    $f(x) = \sin(x)$
    these are cheap to compute.
  - Some functions are very expensive to compute (minutes to hours).
- Ensures that approximation to $I = \int_a^b f(x)\,dx$ is within specified error without needing to input n.
- This minimizes number of times $f(x)$ has to be computed.

Step 1

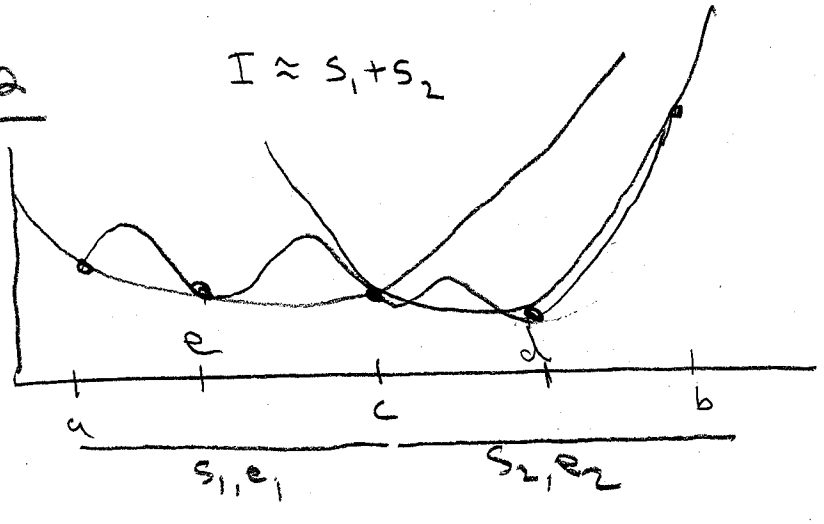
$I \approx S$



- c = midpoint of $[a,b]$
- fit one parabola over entire interval
- probably not very accurate, but you get two things
  $S \cong$ integral approx.
  e = error estimation

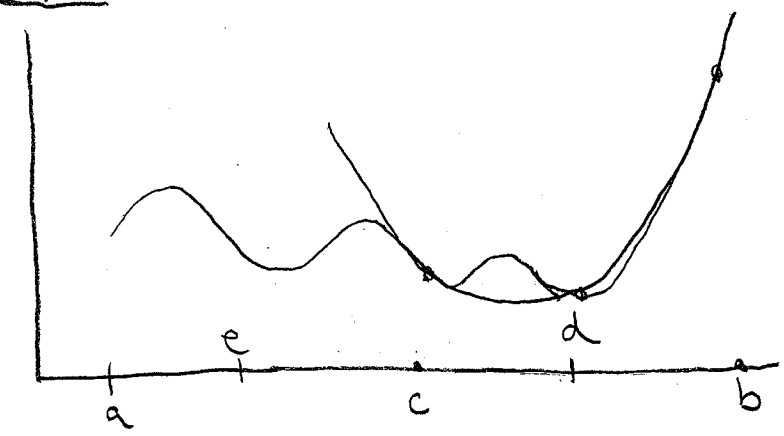
Step 2

$I \approx S_1 + S_2$



- divide $[a,c]$ and $[c,b]$ in half.
- use 2 parabolas
  S_1, e_1 = approx and error for first parabola
  S_2, e_2 = approx and error for second parabola

Step 3



Suppose

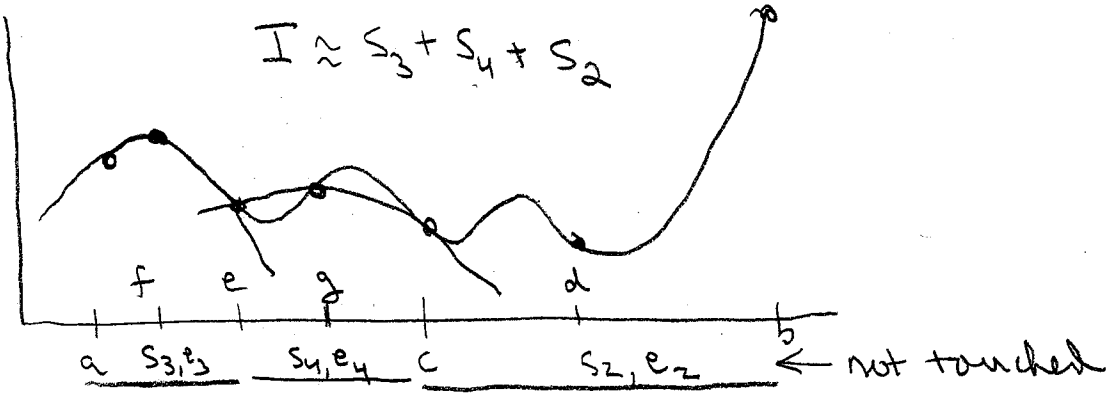
e_2 is much lower than requested tolerance

$\Rightarrow$ "finished" with the contribution on $[c, b]$

e_1 is still greater than requested tolerance.

$\Rightarrow$ apply same principle to $[a, c]$

Step 4



divide $[a,e]$, $[e,c]$ in half

get S_3, e_3

S_4, e_4

Suppose e_3 is less than requested tolerance.

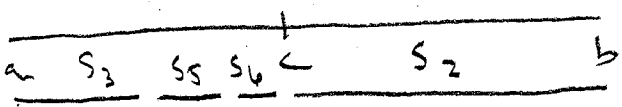
$\Rightarrow$ "finished" with $[a,e]$.

e_4 is still greater than requested tolerance

$\Rightarrow$ apply same idea to $[e,c]$

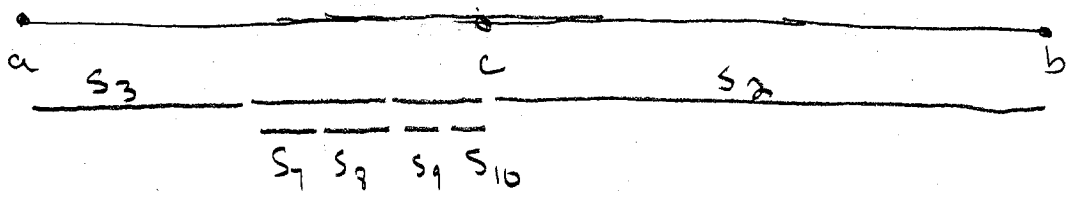
Step 5

done still working



$I \approx S_1 + S_3 + S_5 + S_6$

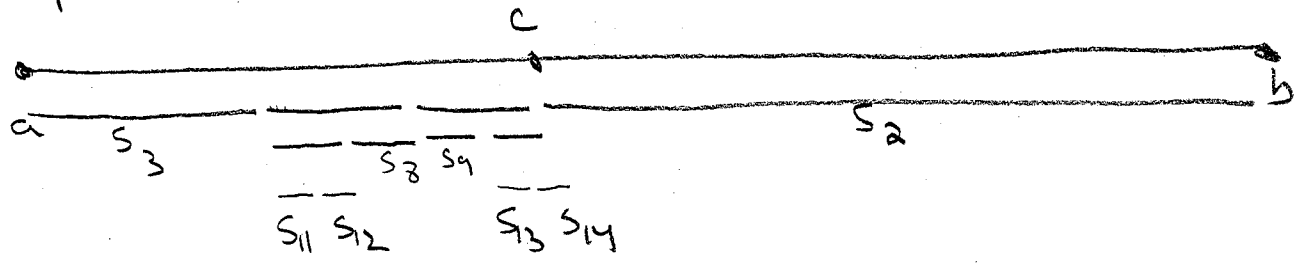
Step 6



$I \approx \underbrace{S_2 + S_3}_{OK} + S_7 + \underbrace{S_8}_{OK} + \underbrace{S_9}_{OK} + S_{10}$

S_7, S_{10} still need work

Step 7



$I \approx \underbrace{S_2 + S_3 + S_8 + S_9}_{OK} + \underbrace{S_{11} + S_{12}}_{OK} + \underbrace{S_{13} + S_{14}}_{\text{still working on this part}}$

- Keep dividing in half until Simpsons approx on each interval is within desired accuracy
- Also, keep track of which x's you have computed $f(x)$ for and reuse these in subsequent calculations rather than recompute them.