

$$1) AD = (5 \times 5) \cdot (5 \times 5) = 3 \times 5$$

OK

$$DA = (5 \times 5) \cdot (3 \times 5)$$

DNE

$$BC = (6 \times 10) \cdot (2 \times 10)$$

DNE

$$BC^T = (6 \times 10) \cdot (10 \times 2) = 6 \times 2$$

$$x^T B = (1 \times 6) \cdot (6 \times 10) = 1 \times 10 \text{ row}$$

$$By = (6 \times 10) \cdot (10 \times 1) = 6 \times 1 \text{ col}$$

$$ADz = (3 \times 5) \cdot (5 \times 5) \cdot (5 \times 1) = 3 \times 1 \text{ col}$$

$$x^T B y = (1 \times 6) \cdot (6 \times 10) \cdot (10 \times 1) = 1 \times 1 \text{ scalar}$$

$$2) \quad x - y = \begin{pmatrix} 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ -1 \end{pmatrix} = \begin{pmatrix} 1-3 \\ 2+1 \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$$

$$x + 2y = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + 2 \begin{pmatrix} 3 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 6 \\ -2 \end{pmatrix} = \begin{pmatrix} 7 \\ 0 \end{pmatrix}$$

$$x^T y = (1 \times 2) \cdot (2 \times 1) = 1 \times 1 \text{ scalar}$$

$$\begin{pmatrix} 1 & 2 \end{pmatrix} \begin{pmatrix} 3 \\ -1 \end{pmatrix} = 1(3) + 2(-1) = 1$$

$$y^T x = (1 \times 2) \cdot (2 \times 1) = \text{scalar}$$

$$\begin{pmatrix} 3 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = 3(1) + 1(-2) = 1$$

$$3) \quad A^T = \begin{pmatrix} -3 & 0 & 1 \\ 1 & 4 & -2 \end{pmatrix}$$

$$Ax = (3 \times 2)(2 \times 1) = 3 \times 1 \text{ col vector}$$

$$= \begin{pmatrix} -3 & 1 \\ 0 & 4 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -3(2) + (1)(1) \\ 0(2) + 4(1) \\ 1(2) + (-2)(1) \end{pmatrix} = \begin{pmatrix} -5 \\ 4 \\ 0 \end{pmatrix}$$

$$A^T x = (2 \times 3)(2 \times 1)$$

DNE

$$y^T A = (1 \times 3)(3 \times 2) = 1 \times 2 \text{ row vector}$$

$$(1 \ -1 \ -2) \begin{pmatrix} -3 & 1 \\ 0 & 4 \\ 1 & -2 \end{pmatrix} = \begin{pmatrix} -5 & 1 \end{pmatrix}$$

$$(1 \ -1 \ -2) \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix} = 1(-3) + (-1)(0) + (-2)(1) = -5$$

$$(1 \ -1 \ -2) \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} = 1(1) + (-1)(4) + (-2)(-2) = 0$$

$$y^T x = (1 \times 2)(2 \times 1) = \text{scalar}$$

$$(3 \ -1) \begin{pmatrix} 1 \\ 2 \end{pmatrix} = 3(1) + (-1)(2) = \boxed{1}$$