

- 1) (2 pts each) Compute an approximate value for each of the integrals below using the trapezoidal rule. In each case, determine the value of n required to make the relative error less than 10^{-6} (note that a relative error of $9.999 \cdot 10^{-6}$ is not less than 10^{-6}). Don't write a program to do this; instead, run your program for various values of n until the required tolerance is met. You don't need to find the exact n that satisfies the tolerance, but you should be close to it. For example, if the n that exactly meets the tolerance is $n = 567$ then $n = 600$ or $n = 650$ would be ok, but $n = 10,000,000$ is not.

You can use the Wolfram Alpha site or something similar to get the exact values of these integrals, but when entering the exact value, do something like in the notes to ensure that your exact value is correct to many digits. In particular, do not do something like

$$\text{exact} = 0.234;$$

This only has 3 digits of accuracy, so the smallest relative error you would be able to compute would be 10^{-3} .

a) $\int_0^{\frac{\pi}{2}} \cos^2(x) dx$

b) $\int_{-1}^1 xe^x dx$

c) $\int_1^3 x \ln(x) dx$

- 2) (2 pts each) Repeat Question 1, but this time use Simpson's Rule. How do the values of n compare between the two methods?
- 3) (3 pts) Consider the integral

$$\int_0^2 y(x) dx$$

where $y(x)$ is the function

$$y(x) = \begin{cases} x & \text{if } 0 \leq x \leq \frac{\pi}{2} \\ 1000x & \text{if } \frac{\pi}{2} < x \leq 2 \end{cases}$$

Without doing any integrations, explain why it would take a very large value of n to make the relative error less than 10^{-6} using the trapezoidal rule. HINT: Graph the function.