Be sure to answer all questions. Your plots should have axis labels and titles.

1) Suppose you have an aluminum rod that is 2 meters long. The rod has a constant cross sectional area A along its length. The temperature (in Kelvin) of the rod as a function of the distance from the left end of the rod is given in the data file `prob1.dat`. The energy (in Joules) contained in the rod at a distance p from the left end is given by the formula

$$E(p) = c\rho A \int_0^p T(x)\,dx.$$

where c and ρ are the specific heat and density of aluminum respectively. Use the values

$$\begin{aligned} c &= 887\frac{J}{kg\,K} \\ \rho &= 2710\frac{kg}{m^3} \\ A &= 0.01m^2. \end{aligned}$$

   a) Why can't you use the trapezoidal or Simpson's rule to perform the integrations needed for this problem?
   b) What is the total energy energy in the rod?
   c) At what point s along the length of the rod is the energy equal to 6×10^6 J?

2) The lapse rate is a meteorological quantity that describes how the air temperature changes with height above the ground and is defined by

$$L = -\frac{dT}{dz}$$

where T is the air temperature in Celsius and z is the height above the Earth in meters. The data file `prob2.dat` contains a table of z versus T (z is the first column). Determine the lapse rate at each value of z. Plot the lapse rate as a function of z (use z as the vertical axis). HINT: Use the `sp_deriv.m` function from Homework 17.

3) An important theorem in mathematics says that if you have 4 points in the (x, y) plane and these points all have different x-coordinates, then there is exactly one cubic function that goes through these points.

   Determine the cubic function that goes through the points $(0, 1), (2, -3), (3, 2)$ and $(5, 4)$. Plot these points and the cubic function that goes through them.

   HINT: Write

$$y = ax^3 + bx^2 + cx + d.$$

   If you substitute the first (x, y) point into this equation, you get a linear equation for a, b, c and d. Do this for all 4 points and you will get a linear system for the coefficients of the cubic.