

- 1) (2 pts each) Compute an approximate value for each of the integrals below using the trapezoidal rule. Use the process towards the end of the trapezoidal rule section of the notes. Write a function to implement the trapezoidal rule and a function that evaluates the integrand. Your main script file should look similar to the one in Section 4.3.

In each case, determine the value of  $n$  required to make the relative error less than  $10^{-6}$  (note that a relative error of something like  $9.94 \cdot 10^{-6}$  is not less than  $10^{-6}$ ). Don't write a program to do this; instead, run your program for various values of  $n$  until the required tolerance is met. You don't need to find the exact  $n$  that satisfies the tolerance, but you should be close to it. For example, if the  $n$  that exactly meets the tolerance is  $n = 567$  then  $n = 600$  or  $n = 650$  would be ok, but  $n = 10,000,000$  is not.

You can use the Wolfram Alpha site or something similar to get the exact values of these integrals, but when entering the exact value, do something like in the notes to ensure that your exact value is correct to many digits. In particular, do not do

```
exact = 0.234;
```

This only has 3 digits of accuracy, so the smallest relative error you would be able to compute would be  $10^{-3}$ .

a)  $\int_0^{\frac{\pi}{2}} \cos^2(x) dx$

b)  $\int_{-1}^1 xe^x dx$

c)  $\int_1^3 x \ln(x) dx$

- 2) (3 pts) Consider the integral

$$\int_0^2 y(x) dx$$

where  $y(x)$  is the function

$$y(x) = \begin{cases} x & \text{if } 0 \leq x \leq \frac{\pi}{2} \\ 1000x & \text{if } \frac{\pi}{2} < x \leq 2 \end{cases}$$

Without doing any integrations, explain why it would take a very large value of  $n$  to make the relative error less than  $10^{-6}$  using the trapezoidal rule. HINT: Graph the function.