

$$1) \quad AD = (3 \times 5) \cdot (5 \times 5) = 3 \times 5$$

OK

$$DA = (5 \times 5) \cdot (3 \times 5)$$

DNE

$$BC = (6 \times 10) \cdot (2 \times 10)$$

DNE

$$BC^T = (6 \times 10) \cdot (10 \times 2) = 6 \times 2$$

$$x^T B = (1 \times 6) \cdot (6 \times 10) = 1 \times 10 \text{ row}$$

$$By = (6 \times 10) \cdot (10 \times 1) = 6 \times 1 \text{ col}$$

$$ADz = (3 \times 5) \cdot (5 \times 5) \cdot (5 \times 1) = 3 \times 1 \text{ col}$$

$$x^T B y = (1 \times 6) \cdot (6 \times 10) \cdot (10 \times 1) = 1 \times 1 \text{ scalar}$$

$$2) \quad x - y = \begin{pmatrix} 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ -1 \end{pmatrix} = \begin{pmatrix} 1-3 \\ 2+1 \end{pmatrix} = \boxed{\begin{pmatrix} -2 \\ 3 \end{pmatrix}}$$

$$x + 2y = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + 2 \begin{pmatrix} 3 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 6 \\ -2 \end{pmatrix} = \boxed{\begin{pmatrix} 7 \\ 0 \end{pmatrix}}$$

$$x^T y = (1 \times 2) \cdot (2 \times 1) = 1 \times 1 \text{ scalar}$$

$$\begin{pmatrix} 1 & 2 \end{pmatrix} \begin{pmatrix} 3 \\ -1 \end{pmatrix} = 1(3) + 2(-1) = \boxed{1}$$

$$y^T x = (1 \times 2) \cdot (2 \times 1) = \text{scalar}$$

$$\begin{pmatrix} 3 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = 3(1) + 1(-2) = \boxed{1}$$

$$3) \quad A^T = \begin{pmatrix} -3 & 0 & 1 \\ 1 & 4 & -2 \end{pmatrix}$$

$$Ax = (3 \times 2) \underset{\substack{\uparrow \\ \text{OK}}}{(2 \times 1)} = 3 \times 1 \text{ col vector}$$

$$= \begin{pmatrix} -3 & 1 \\ 0 & 4 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -3(2) + (1)(1) \\ 0(2) + 4(1) \\ 1(2) + (-2)(1) \end{pmatrix} = \boxed{\begin{pmatrix} -5 \\ 4 \\ 0 \end{pmatrix}}$$

$$A^T x = (2 \times 3) \underset{\substack{\uparrow \\ \text{DNE}}}{(2 \times 1)}$$

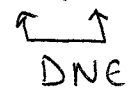
$$y^T A = (1 \times 3) \underset{\substack{\uparrow \\ \text{OK}}}{(3 \times 2)} = 1 \times 2 \text{ row vector}$$

$$(1 \ -1 \ -2) \left(\begin{array}{c|c} -3 & 1 \\ 0 & 4 \\ 1 & -2 \end{array} \right) = \boxed{\begin{pmatrix} -5 & 1 \end{pmatrix}}$$

$$(1 \ -1 \ -2) \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix} = 1(-3) + (-1)(0) + (-2)(1) = -5$$

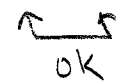
$$(1 \ -1 \ -2) \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} = 1(1) + (-1)(4) + (-2)(-2) = 0$$

$$y^T B = (1 \times 3)(2 \times 3)$$



DNE

$$AB = (3 \times 2)(2 \times 3) = 3 \times 3$$



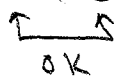
OK

$$\begin{pmatrix} -3 & 1 \\ 0 & 4 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ -1 & 2 & -1 \end{pmatrix}$$

$$= \begin{bmatrix} -3(1) + 1(-1) & -3(2) + 1(2) & -3(3) + 1(-1) \\ 0(1) + 4(-1) & 0(2) + 4(2) & 0(3) + 4(-1) \\ 1(1) + (-2)(-1) & 1(2) + (-2)(2) & 1(3) + (-2)(-1) \end{bmatrix}$$

$$= \begin{bmatrix} -4 & -4 & -10 \\ -4 & 8 & -4 \\ 3 & -2 & 5 \end{bmatrix}$$

$$BA = (2 \times 3)(3 \times 2) = 2 \times 2$$



OK

$$\begin{pmatrix} 1 & 2 & 3 \\ -1 & 2 & -1 \end{pmatrix} \begin{pmatrix} -3 & 1 \\ 0 & 4 \\ 1 & -2 \end{pmatrix}$$

$$= \begin{bmatrix} 1(-3) + 2(0) + 3(1) & 1(1) + 2(4) + 3(-2) \\ -1(-3) + 2(0) + (-1)(1) & -1(1) + 2(4) + (-1)(-2) \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 3 \\ 2 & 9 \end{bmatrix}$$

4)

$$\|x\|_1 = |2| + |1| = 3$$

$$\|x\|_\infty = \max\{2, 1\} = 2$$

$$\|x\|_2 = \sqrt{2^2 + 1^2} = \sqrt{5}$$

$$\|A\|_1 = \max\{4, 7\} = 7$$

$$\|A\|_\infty = \max\{4, 4.3\} = 4$$

$$\|A\|_F = \sqrt{(-3)^2 + 1^2 + 0^2 + 4^2 + 1^2 + (-2)^2} = \sqrt{31}$$

5)

$$x^T x = \sum_{i=1}^n x_i x_i = \sum_{i=1}^n x_i^2$$

$$\|x\|_2 = \sqrt{\sum_{i=1}^n x_i^2} = \sqrt{x^T x}$$

$$\} \Rightarrow \|x\|_2^2 = x^T x$$