For the problems below, remember to answer all the questions being posed. Print out the requested quantities and graphs. Hand in all of the codes that you write.

1) (6 pts) Solve the initial value problem

$$\begin{aligned} x^4y'''' + 2x^3y''' + x^2y'' - xy' + 2y &= ye^{-x}, \quad x \in [1,5] \\ y(1) &= 3 \\ y'(1) &= -2 \\ y''(1) &= 1 \\ y'''(1) &= 0 \end{aligned}$$

   Plot the values of y, y', y'', y''' and y'''' on a common axis.

2) (4 pts) The simple, undamped, unforced pendulum is described by the equation

$$\begin{aligned} \theta'' + \frac{g}{L}\theta &= 0 \\ \theta(0) &= \theta_0 \\ \theta'(0) &= \theta_1. \end{aligned}$$

   Here, θ represents the angular displacement of a bob of mass m suspended by string of length L from an immobile support. θ' is the angular velocity of the bob and θ'' is the angular acceleration.

   Solve this system assuming $g = 9.81\text{m/s}^2, L = 2\,\text{m}, \theta_0 = \frac{\pi}{3}$ rad and $\theta_1 = 0\,\text{rad/s}$. Plot values of θ, θ' and θ'' on a common axis.

   Also, create a *phase plane* plot by plotting θ on the x-axis and θ' on the y-axis. What shape does the phase plane plot have? Use the command `axis equal` to be sure of your answer.

3) (8 pts) In Problem 2, you should have observed that θ is a periodic function. Devise and describe a computational technique for determining the period of θ. There a several ways to do this using the functions we have written.

4) (4 pts) The real, undamped, unforced pendulum is described by the equation

$$\begin{aligned} \theta'' + \frac{g}{L}\sin\theta &= 0 \\ \theta(0) &= \theta_0 \\ \theta'(0) &= \theta_1. \end{aligned}$$

   Solve this system for the same set of parameters as in Problem 2. Create the same plots also. Does θ appear to be periodic for the real pendulum? If so, compute the period and compare it to the period you obtained in Question 3. Does the phase plane plot have the same shape?