

- 1) (2 pts) Suppose you have the following vectors/matrices with indicated dimensions:

$$A(= 3 \times 5); \quad B(= 6 \times 10); \quad C(= 2 \times 10); \quad D(= 5 \times 5);$$

$$x(= 6 \times 1); \quad y(= 10 \times 1); \quad z(= 5 \times 1).$$

What are the resulting dimensions of the following calculations:

$$AD, DA, BC, BC^T, x^T B, By, ADz, x^T B y?$$

If you cannot perform an operation, state why not. Also, if the result is a vector, state whether it is a row vector or column vector.

- 2) (4 pts) Given the vectors

$$x = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \quad y = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

compute the following:  $x - y$ ,  $x + 2y$ ,  $x^T y$ ,  $y^T x$ .

- 3) (11 pts) Given the quantities

$$A = \begin{pmatrix} -3 & 1 \\ 0 & 4 \\ 1 & -2 \end{pmatrix}; \quad B = \begin{pmatrix} 1 & 2 & 3 \\ -1 & 2 & -1 \end{pmatrix}; \quad x = \begin{pmatrix} 2 \\ 1 \end{pmatrix}; \quad y = \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$$

compute the following:  $A^T$ ,  $Ax$ ,  $A^T x$ ,  $y^T A$ ,  $y^T B$ ,  $AB$ ,  $BA$ . If a quantity cannot be computed, state the reason why.

- 4) (6 pts) For  $A$  and  $x$  as defined in Problem 4, compute  $\|x\|_1$ ,  $\|x\|_2$ ,  $\|x\|_\infty$ ,  $\|A\|_1$ ,  $\|A\|_\infty$ ,  $\|A\|_F$ .

- 5) (2 pts) Prove that

$$\|x\|_2^2 = x^T x.$$