ESC 251 **Project 1** **Due: Nov. 30**

You may not discuss this project with anyone but me. A score of zero will be given to students that do not follow this instruction.

This example is modified from material in *Mathematics in Medicine and the Life Sciences*, Hoppensteadt and Peskin, 1992. The Jacob-Monod model is used to describe the growth of populations in the presence of a finite quantity of food. It is defined by the equations

$$\begin{aligned}\frac{dx}{dt} &= \frac{Vy}{K+y}x \\ \frac{dy}{dt} &= -\frac{1}{Y}\frac{Vy}{K+y}x\end{aligned}$$

where

- x = Concentration of bacteria in parts per million
- y = Food supply
- V = Rate at which x consumes y
- K = Saturation constant
- Y = Yield of x per unit of y

Part 1

Using the model equations and definitions above, do the following (by hand):

1) Show that $C = x + Yy$ is a constant.
2) Show that x and y are linearly related.
3) What is $\lim_{t\to\infty} y$? (just think about what y is physically).
4) Use 1) and 3) to find $\lim_{t\to\infty} x$.

Part 2

Set up the MATLAB files necessary to solve this set of equations. Run your model for the values $V = 5$, $K = 10$ and $Y = 100$ on the time interval $t \in [0, 3]$ with the initial conditions $x(0) = 100, y(0) = 10$. Do your results confirm the conclusions above? You should plot x and y in both the time and phase spaces in addition to any plots necessary to verify the conclusions.

Part 3

Once your model is working, modify your driver script file to do the following:

1) Create a plot of x versus time for V values of $V = 5, 6, 7, 8, 9, 10$. Use the same values for the other quantities as in Part 2.
2) For each value of V above, compute the value t_{critical}. This is the time at which x reaches 99% of its limiting value.
3) Fit a power function to the resulting data for V vs. t_{critical}. See the attached PDF for details on performing power law fits in MATLAB.
4) Use the result from 3) above to determine the value of V necessary to obtain $t_{\text{critical}} = 1$.

5) Rerun the model for the value of V you found in part 4). How close is t_{critical} to 1?

6) Given the nature of the model, how far beyond t_{critical} would you expect the model to be valid?

You should hand in a report that clearly answers the questions in the Part 1 in addition to a discussion of the results obtained from the numerical work in Parts 2 and 3. Include figures and/or tables in your discussion as appropriate. Note that there are some things you need to determine in order to do this problem that are not specifically indicated above. Your report should describe how you fill in these blanks.