HW 4 Soln

6) all vectors are 3×1

a) $X * w \rightarrow$ can't do

b) $w * w \rightarrow$ can't do

c) $w' * w \rightarrow \begin{pmatrix} 4 & 2 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix} = 4 \cdot 4 + 2 \cdot 2 + 1 \cdot 1 = 21$

d) $y' * z \rightarrow \begin{pmatrix} 2 & -3 & 1 \end{pmatrix} \begin{pmatrix} i \\ -i \\ 3i \end{pmatrix} = 2i + 3i + 3i = 8i$

e) $y * z \rightarrow$ can't do

f) $y * y' = (3 \times 1)(1 \times 3) = 3 \times 3$ Each dot product is a single term

$$\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} \begin{pmatrix} 2 & 1 & -3 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 4 & -6 & 2 \\ -6 & 9 & -3 \\ 2 & -3 & 1 \end{pmatrix}$$

g) $y .* w = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}$ dot multiplication multiply corresponding entries

$$= \begin{pmatrix} 8 \\ -6 \\ 1 \end{pmatrix}$$

h) $y .* w'$ → can't do b/c y is a column and w' is a row

i) $x .* z = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} i \\ -i \\ 3i \end{pmatrix} = \begin{pmatrix} i \\ 0 \\ 3i \end{pmatrix}$

j) $x ./ y = \dfrac{\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}}{\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}}$ divide corresponding elements

$= \begin{pmatrix} 1/2 \\ 0 \\ 1 \end{pmatrix}$

7.

a) $A = 3\times3$, $x = 3\times1$ $\quad A*x = (3\times3)(3\times1) = 3\times1$

$$A*x = \begin{pmatrix} -1 & 3 & 4 \\ 6 & -2 & -9 \\ 3 & 1 & 0 \end{pmatrix}\begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix} = \begin{pmatrix} -1(3)+3(-2)+4(0) \\ 6(3)-2(-2)-9(0) \\ 3(3)+1(-2)+0\cdot0 \end{pmatrix}$$

$$= \begin{pmatrix} -9 \\ 22 \\ 7 \end{pmatrix}$$

b) $x*A = (3\times1)(3\times3) \rightarrow$ DNE

c) $A*y = (3\times3)(1\times3) \rightarrow$ DNE

d) $y*A = (1\times3)(3\times3) \rightarrow 1\times3$ row vector

$$= \begin{pmatrix} -3 & 4 & 1 \end{pmatrix}\left(\begin{array}{c|c|c} -1 & 3 & 4 \\ 6 & -2 & -9 \\ 3 & 1 & 0 \end{array}\right) = \begin{bmatrix} -3(-1)+4(6)+1(3) & -3(3)+4(-2)+1(1) & \\ & & -3(4)+4(-9)+1(0) \end{bmatrix}$$

$$= [30 \quad -16 \quad -48]$$

e) $A' * x = (3\times3)(3\times1) = 3\times1$

$$= \begin{pmatrix} -1 & 6 & 3 \\ 3 & -2 & 1 \\ 4 & -9 & 0 \end{pmatrix}\begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix} = \begin{pmatrix} -1(3)+6(-2)+3(0) \\ 3(3)+(-2)(-2)+1(-2) \\ 4(3)+(-9)(-2)+0\cdot0 \end{pmatrix}$$

$$= \begin{pmatrix} -15 \\ 13 \\ 30 \end{pmatrix}$$

f) $A * y' = (3\times3)(3\times1) = 3\times1$

$$\begin{pmatrix} -1 & 3 & 4 \\ 6 & -2 & -9 \\ 3 & 1 & 0 \end{pmatrix}\begin{pmatrix} -3 \\ 4 \\ 1 \end{pmatrix} = \begin{pmatrix} (-1)(-3)+3(4)+4(1) \\ 6(-3)+(-2)(4)+(-9)(1) \\ 3(-3)+1(4)+0\cdot1 \end{pmatrix}$$

$$= \begin{pmatrix} 19 \\ -35 \\ -5 \end{pmatrix}$$

g) $y' * A * x = (3\times1)(3\times3)(3\times1)$ DNE

(3) a) $A * B = (3\times 3)(3\times 2) = 3\times 2$

$$\left(\begin{array}{ccc} 0 & 0 & 1 \\ \hline 1 & -1 & -2 \\ \hline -1 & 6 & 3 \end{array}\right)\left(\begin{array}{c|c} 2 & -1 \\ 0 & 3 \\ \hline -2 & 2 \end{array}\right) = \begin{pmatrix} \boxed{1} & \boxed{2} \\ \boxed{3} & \boxed{4} \\ \boxed{5} & \boxed{6} \end{pmatrix} = \begin{pmatrix} -2 & 2 \\ 6 & -8 \\ -8 & 25 \end{pmatrix}$$

$\boxed{1} = 0\cdot 2 + 0\cdot 0 + 1(-2) = -2$

$\boxed{2} = 0\cdot(-1) + 0\cdot 3 + 1(2) = 2$

$\boxed{3} = 1(2) + (-1)(0) + (-2)(-2) = 6$

$\boxed{4} = 1(-1) + (-1)(3) + (-2)(2) = -8$

$\boxed{5} = -1(2) + 6(0) + 3(-2) = -8$

$\boxed{6} = -1(-1) + 6(3) + 3(2) = 25$

b) $A * C = (3\times 3)(2\times 3) = DNE$

c) $C * A = (2\times 3)(3\times 3) = 2\times 3$

$$\left(\begin{array}{ccc} 0 & 2 & -1 \\ \hline -3 & 0 & 1 \end{array}\right)\left(\begin{array}{c|c|c} 0 & 0 & 1 \\ 1 & -1 & -2 \\ \hline -1 & 6 & 3 \end{array}\right) = \begin{pmatrix} \boxed{1} & \boxed{2} & \boxed{3} \\ \boxed{4} & \boxed{5} & \boxed{6} \end{pmatrix} = \begin{pmatrix} 3 & -8 & -7 \\ -1 & 6 & 0 \end{pmatrix}$$

$\boxed{1} = 0\cdot 0 + 2(1) + (-1)(-1) = 3$

$\boxed{2} = 0\cdot 0 + 2(-1) + (-1)(6) = -8$

$\boxed{3} = 0\cdot 1 + 2(-2) + (-1)(3) = -7$

$\boxed{4} = -3(0) + 0(1) + 1(-1) = -1$

$\boxed{5} = -3(0) + 0(-1) + 1(6) = 6$

$\boxed{6} = -3(1) + 0(-2) + 1(3) = 0$

9) a) $y = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$

$z = y.\wedge 3$ → cube each element of y

$z = \begin{pmatrix} 2^3 \\ (-3)^3 \\ (1)^3 \end{pmatrix} = \begin{pmatrix} 8 \\ -27 \\ 1 \end{pmatrix}$

b) $z = \sqrt{y .* y}$

$y .* y = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 9 \\ 1 \end{pmatrix}$

$\sqrt{\begin{pmatrix} 4 \\ 9 \\ 1 \end{pmatrix}} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$

$z = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$

10) $D = \begin{pmatrix} 2 & 4 \\ 3 & 6 \end{pmatrix}$

a) $D^2 = D * D = \left(\begin{matrix} 2 & 4 \\ 3 & 6 \end{matrix}\right)\left(\begin{matrix} 2 & 4 \\ 3 & 6 \end{matrix}\right) = \begin{pmatrix} \boxed{1} & \boxed{2} \\ \boxed{3} & \boxed{4} \end{pmatrix}$

$\boxed{1} = 2(2) + 4(3) = 16$

$\boxed{2} = 2(4) + 4(6) = 32$

$\boxed{3} = 3(2) + 6(3) = 24$

$\boxed{4} = 3(4) + 6(6) = 48$

$= \begin{pmatrix} 16 & 32 \\ 24 & 48 \end{pmatrix}$

b) $D.^2$ = Square each element of D

$= \begin{pmatrix} 4 & 16 \\ 9 & 36 \end{pmatrix}$

11) Euler Identity: $e^{i\pi} + 1 = 0$

$e^{i\pi} + 1 = 0$

$-1 = e^{i\pi}$

$(-1)^{1/3} = (e^{i\pi})^{1/3} = e^{i\pi/3} = \cos(\pi/3) + i\sin(\pi/3)$

$= 1/2 + \frac{\sqrt{3}}{2}i$