Inverse functions

$$e^{\ln(x)} = ?$$

$$\ln(e^x) = ?$$

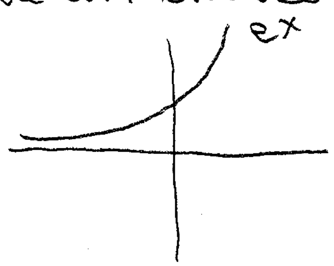
$$e^{\ln(x)} = x \quad \text{if } x > 0$$

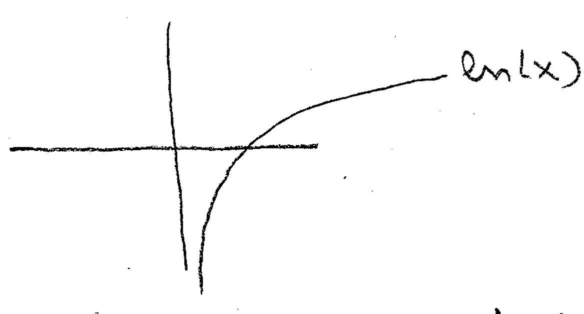
$$\ln(e^x) = x \quad \text{for all } x$$

- e^x, $\ln(x)$ are inverse functions. Action of one will undo the action of the other.
- $\log_{10}(x)$, 10^x inverse functions
- x^2, $\sqrt{x}$ are inverse functions

$$\sqrt{x^2} = x, \quad (\sqrt{x})^2 = x \quad \text{if } x \text{ is positive}$$

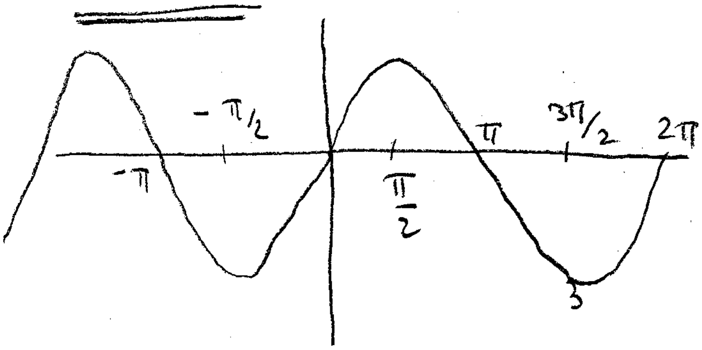
- A function needs to pass the horizontal line test in order to have an inverse



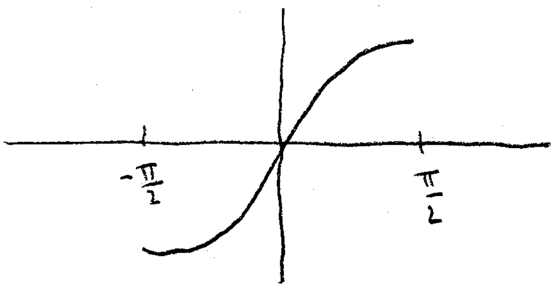


- often times, the domain and/or range needs to be restricted in order to enforce the horizontal line test.

Sin (x)



- Does not pass the horizontal line test.
- It will pass the test if we apply some limits



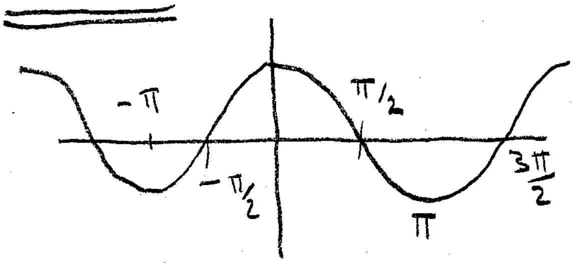
- $\sin^{-1}(x) = \arcsin(x)$ = angle whose sine is x

  Domain = $[-1, 1]$

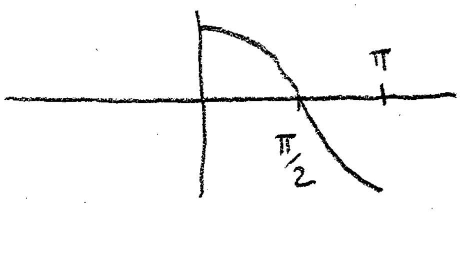
  Range = $[-\pi/2, \pi/2]$

  → regardless of input value, output is always in $[-\pi/2, \pi/2]$

cos (x)



- Also does not pass horizontal line test.
- It will if we apply some limits



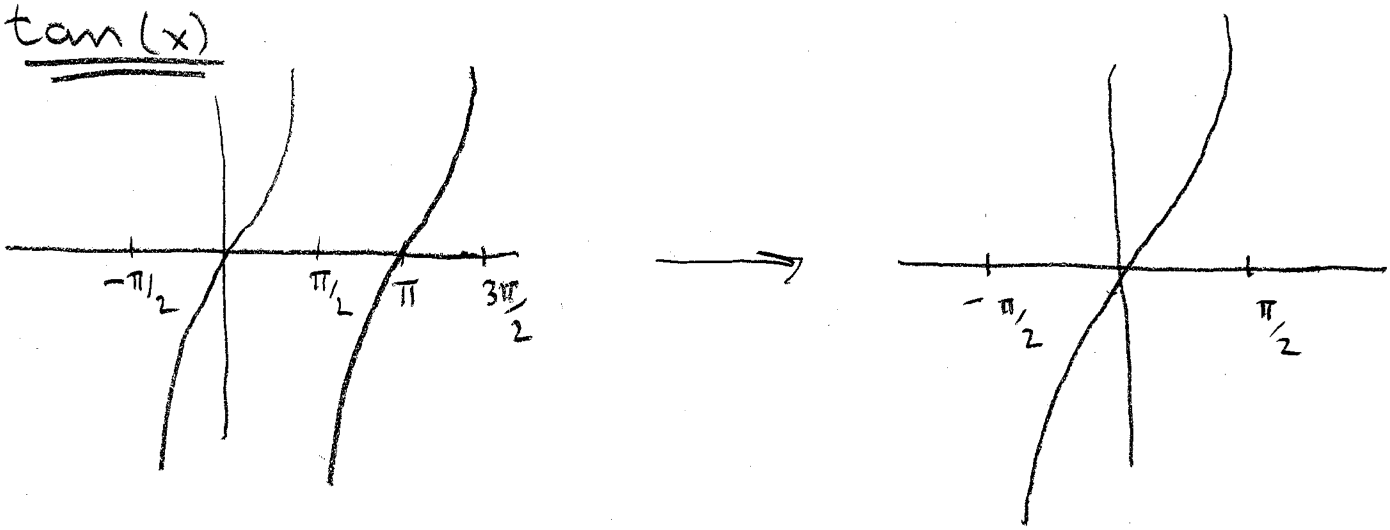
- $\cos^{-1}(x) = \arccos(x) =$ angle whose cosine is x

  Domain $= [-1, 1]$

  Range $= [0, \pi]$

$\rightarrow$ output is always an angle in $[0, \pi]$

tan(x)



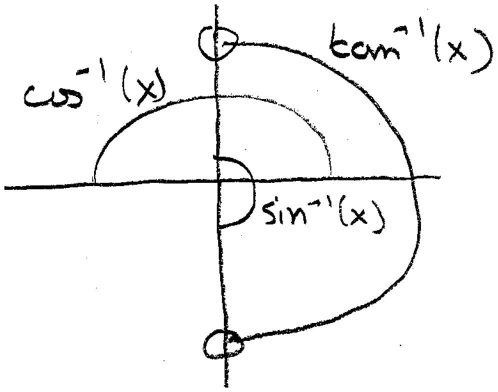
- Restrict to $(-\pi/2, \pi/2)$

- $\tan^{-1}(x) = \arctan(x) =$ angle whose tangent is x

  Domain $\Rightarrow$ any x in $(-\infty, \infty)$

  Range $= (-\pi/2, \pi/2)$

$\rightarrow$ output is always an angle in $(-\pi/2, \pi/2)$



HW8, P4

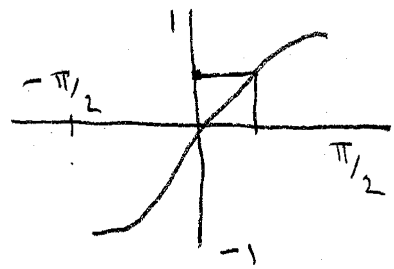
- Want to compute the sine of an angle, then compute the inverse sine of the sine to see if you get the original angle back

Does $\sin^{-1}(\sin(x)) = x$?

$x = 30^\circ \ (= \pi/6)$

$\sin(\pi/6) = 1/2$

$\sin^{-1}(1/2) = \pi/6$

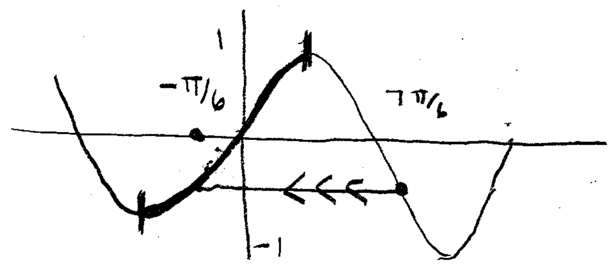


True

$x = 210^\circ \ (= 7\pi/6)$

$\sin(7\pi/6) = -1/2$

$\sin^{-1}(-1/2) = -\pi/6$ not true



$\sin^{-1}(\sin(x)) = x$ only if x is a principle angle in $[-\pi/2, \pi/2]$

otherwise we get something equivalent, but not exactly what we started with

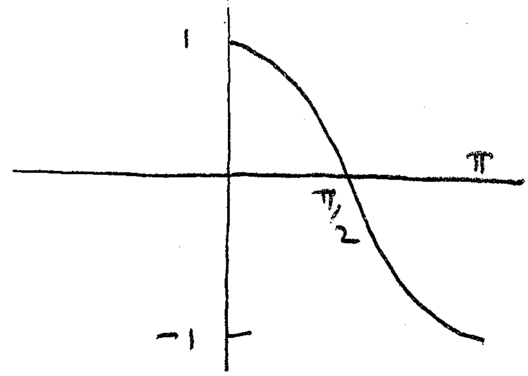
note $\sin(\sin^{-1}(x)) = x$ true for all x

- Does $\cos^{-1}(\cos(x)) = x$?

$\underline{\underline{30^\circ}}$ $(= \pi/6)$

$\cos(\pi/6) = \frac{\sqrt{3}}{2}$

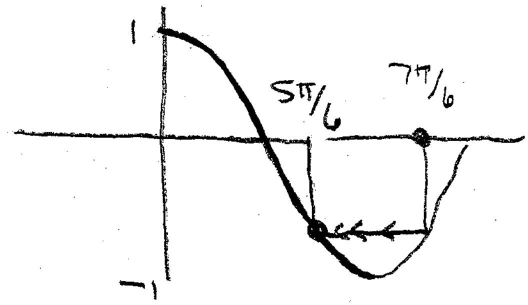
$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \pi/6$ True



$\underline{\underline{210^\circ}}$ $(= 7\pi/6)$

$\cos(7\pi/6) = -\frac{\sqrt{3}}{2}$

$\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$ not true



$\cos^{-1}(\cos(x)) = x$ only if x is a principle angle in $[0, \pi]$

$\cos(\cos^{-1}(x)) = x$ true for all x

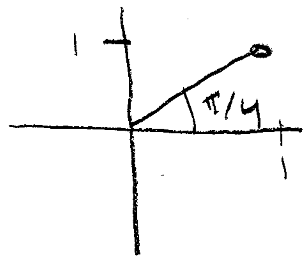
- Does $\tan^{-1}(\tan(x)) = x$

only if x is a principle angle in $(-\pi/2, \pi/2)$

Problem (HW9, P5)

Sometimes we need the angle to be the actual angle (like when converting to polar coordinates)

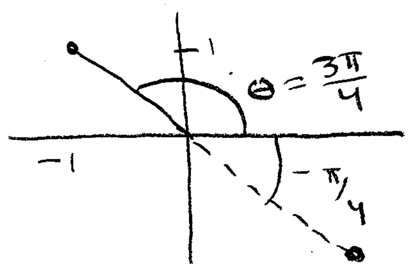
$(x,y) = (1,1)$



$r = \sqrt{1^2+1^2} = \sqrt{2}$

$\theta = \tan^{-1}\left(\frac{1}{1}\right) = \tan^{-1}(1) = \pi/4$

$(x,y) = (-1,1)$



$r = \sqrt{1^2+1^2} = \sqrt{2}$

$\theta = \tan^{-1}\left(\frac{1}{-1}\right) = \tan^{-1}(-1) = -\pi/4$

θ not correct. we need $\theta = 3\pi/4$, instead we get the equivalent quadrant IV point.

- ATAN(x) = principle arctan
- ATAN2(y,x) = 4 quadrant arctan

$= 4$ quadrant $\tan^{-1}\left(\frac{y}{x}\right)$

ATAN2 returns an angle in $[-\pi, \pi]$

HW9, P3

$y = ACOS(2.3d\emptyset)$

This is asking for the principle angle whose cosine is 2.3d∅ radians

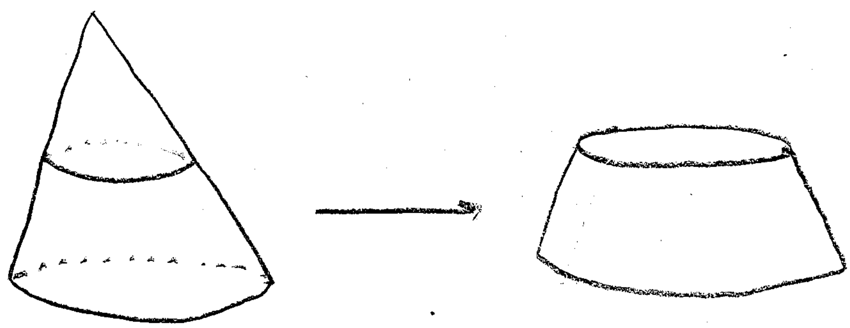
→ This angle does not exist, so the compiler flags this statement with an error

$y = ATAN(2.3d\emptyset)$

This is asking for the principle angle whose tangent is 2.3d∅.

→ This is fine, so the compiler gives a value

HW8, P5

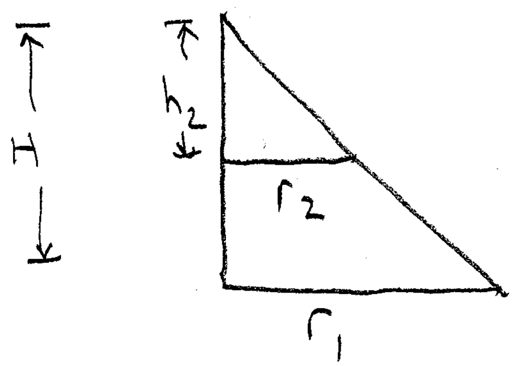


Volume = Volume large cone − Volume small cone

H = height of large cone

r_1 = base of large cone

r_2 = base of small cone



$$\frac{h_2}{H} = \frac{r_2}{r_1} \rightarrow h_2 = \frac{H r_2}{r_1}$$

$$\begin{aligned}\text{Volume} &= \tfrac{1}{3}\pi r_1^2 H - \tfrac{1}{3}\pi r_2^2 h_2 \\ &= \tfrac{1}{3}\pi r_1^2 H - \tfrac{1}{3}\pi r_2^2 \cdot \frac{H r_2}{r_1} \\ &= \tfrac{1}{3}\pi H \left(r_1^2 - r_2^3 / r_1 \right)\end{aligned}$$

Key points so far

* use syntax highlighting (run setnote before doing anything else)

* watch for integer division ($1/3 = 0$)

* add 'd' descriptor to constants that have digits after the decimal point

  4.91 → 4.91d0

  0.326 → 0.326d0

  0.00024 → 0.00024d0 <u>or</u> 2.4d-4