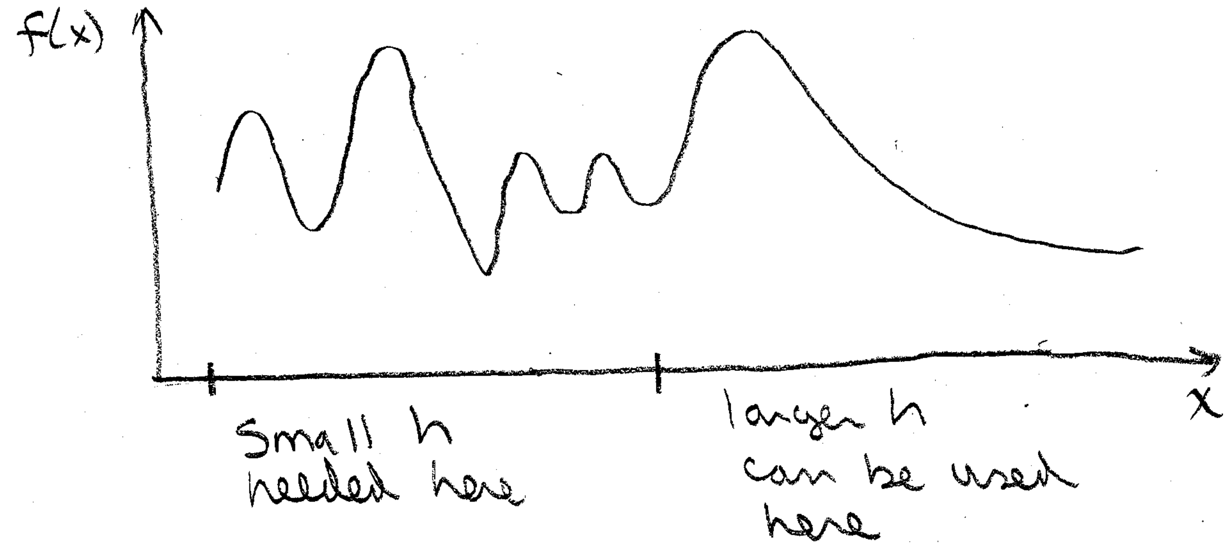
To this point, we have assumed that the points in the table are equally spaced in the x-coordinate. This is not always the case. Sometimes a variable spacing of the x-coordinate leads to better computational efficiency.

Ex: Trapezoidal Rule



In the first part of the graph, f(x) varies rapidly. This requires a small h to ensure the behavior of the function is accurately resolved.

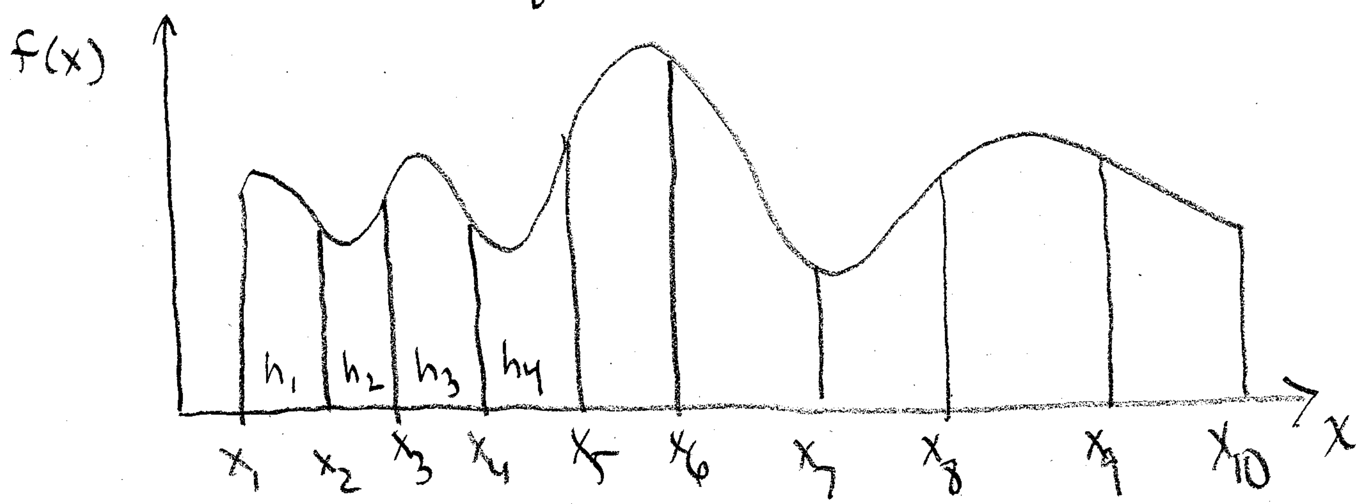
In the latter portion of the graph, f(x) varies slowly, so a larger h can be used.

If h is a constant value, then you need a small h everywhere to resolve the first portion of the graph.

An adaptive trapezoidal method will automatically choose the h necessary to obtain a requested degree of accuracy.

As a result, the method will automatically select smaller h values where needed.

How this is done is beyond the scope of this course, but we can pretend this has been done and modify the trapezoidal rule as necessary



Here, the width of each panel can vary from one panel to the next.

From the figure, it can be seen that

$$h_i = x_{i+1} - x_i$$

The area of the first panel will be

$$A_1 = \frac{h_1}{2}\left(f(x_1)+f(x_2)\right)$$

$$= \frac{(x_2-x_1)}{2}\left(f(x_1)+f(x_2)\right)$$

Similarly

$$A_2 = \frac{(x_3-x_2)}{2}\left(f(x_2)+f(x_3)\right)$$

In general,

$$A_i = \frac{(x_{i+1}-x_i)}{2}\left(f(x_i)+f(x_{i+1})\right)$$

And the trapezoidal method becomes

$$\int_a^b f(x)\,dx \approx \sum_{i=1}^{n} \frac{(x_{i+1}-x_i)}{2}\left(f(x_i)+f(x_{i+1})\right)$$