Summary of Results so far

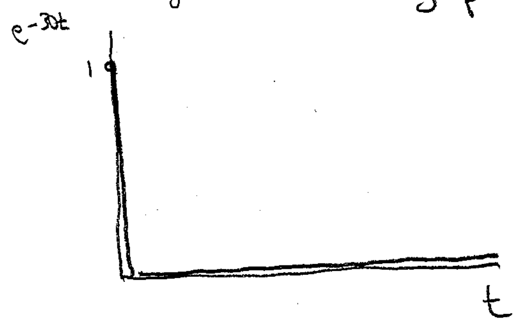
	Euler	RK4
Problem 1	16-32 Trillion	81
Problem 2	×	50
Problem 3		1800

?

Problem 3 seems to take an unusual number of steps.

$$\frac{2673\,e^{-30t} + \sin(t) + 30\cos(t)}{901} = y_{exact}$$

e^{-30t} goes to 0 very quickly



$$y' = \underbrace{-30y}_{\to 0 \text{ quickly}} + \cos(t)$$

Solution starts looking like sin(t) ⇒ reasonable n value should work.

unfortunately, this is not what happens.

The phenomena that is being observed here happens for very non obvious reasons

The underlying cause is stability

< leave off lots of theory >

- RK4 produces an approximate solution
- substitute exact solution into RK4 method
- Resulting expression is the error
- This error expression will have a term like

$$(c_1 h - c_2)^i \quad \text{for some constants } c_1, c_2$$

- in limit as $i \to \infty$ (or as i gets large)

$$(c_1 h - c_2)^i \to \infty \quad \text{if} \quad c_1 h - c_2 > 1$$
$$(c_1 h - c_2)^i \to 0 \quad \text{if} \quad c_1 h - c_2 < 1$$

Solution won't converge unless

$$c_1 h - c_2 < 1$$

$$c_1 h < 1 + c_2$$

$$h < \frac{1 + c_2}{c_1}$$

<u>Problem</u> For this example problem, c_1 is large

$\Rightarrow$ h is small

$\Rightarrow$ n is large

This is a <u>stiff</u> problem. It occurs any time the solution contains a term like e^{-at} for $a > 6$ or so, For this problem, e^{-30t} is very stiff.

Why this is important

1) RK 4 won't work

2) This type of problem is common in Chemical reactions

<u>Terminology</u>

Euler and RK4 are <u>one step methods</u>

→ only y_i is needed to produce y_{i+1}

Euler and RK4 are <u>Explicit methods</u>

→ The formulas are both of the type
y_{i+1} = Something involving t_i, y_i

Look at another Euler method

$$y_{i+1} = y_i + hf(t_{i+1}, y_{i+1})$$

This is now evaluated at t_{i+1}, y_{i+1} instead of t_i, y_i

This is the backwards (or implicit) Euler method

Now the unknown quantity (y_{i+1}) shows up on both sides of the equation

Implicit methods are much more stable than explicit methods $\longrightarrow$ good for stiff problems

Problem

$$y_{i+1} = y_i + h f(t_{i+1}, y_{i+1})$$

we don't know y_{i+1}, so how can we compute $f(t_{i+1}, y_{i+1})$?

Solution

Make a really good guess

A really good guess would be

$$y^*_{i+1} = y_i + h f(t_i, y_i)$$

This is called a predictor corrector method

$$\text{temp} = y_i + h f(t_i, y_i) \quad \text{Explicit Euler predictor}$$

$$y_{i+1} = y_i + h(t_{i+1}, \text{temp}) \quad \text{Implicit Euler corrector}$$

So....

we need an implicit method with an accuracy similar to RK4

one choice: 4th order Adams predictor-corrector

$$temp = y_i + \frac{h}{24}\Big(55f(t_i,y_i) - 59f(t_{i-1},y_{i-1}) + 37f(t_{i-2},y_{i-2}) - 9f(t_{i-3},y_{i-3})\Big)$$

$$y_{i+1} = y_i + \frac{h}{24}\Big(9f(t_{i+1},temp) + 19f(t_i,y_i) - 5f(t_{i-1},f_{i-1}) + f(t_{i-2},y_{i-2})\Big)$$

Notes

1) unlike RK4 and Euler, this is a multistep method. Information about $f(t_i,y_i)$, $f(t_{i-1},y_{i-1})$, $f(t_{i-2},y_{i-2})$ and $f(t_{i-3},y_{i-3})$ is needed

2) How to get started? We need more than the initial condition because we need 3 previous values to get each new value. Use RK4 to get started

3) We need to save the evaluations of $f(t,y)$

Denote $f_i = f(t_i,y_i)$

Outline

```
Set   t(1) = 0
      y(1) = initial cond
      f(1) = rhsfun(t(1),y(1))

  compute
      y(2), f(2)
      y(3), f(3)
      y(4), f(4)    using RK4

  DO i = 4,n
      compute y(i+1) using predictor corrector
  ENDDO
```