(15 pts) Recall that the standard 4th order Runge-Kutta method for solving a first order initial value problem is given by

```
y(0) = y0

for i = 0,1,....
   k1 = f(t(i),y(i))
   k2 = f(t(i)+h/2,y(i)+h*k1/2)
   k3 = f(t(i)+h/2,y(i)+h*k2/2)
   k4 = f(t(i)+h,y(i)+h*k3)
   y(i+1) = y(i) + h*(k1 + 2*k2 + 2*k3 + k4)/6
```

Write a main program and accompanying subroutines/functions that will implement this method for solving the first order initial value problem

$$y'(t) = f(t, y(t)), \quad y(0) = y_0, \quad \text{on} \quad t \in [0, b].$$

Use your program to solve the following problems. In each case, determine the value of n necessary for the 2-norm of the error vector to be less than 10^{-6}. Use separation of variables or the Wolfram Alpha site to determine the exact solutions.

a)
$$y' = -ty, \quad y(0) = 2, \quad \text{on} \quad t \in [0, 4].$$

How does the value of n for this problem compare to the one for Euler's method?

b)
$$y' = -y\sin(t), \quad y(0) = 3, \quad \text{on} \quad t \in [0, 4].$$

c)
$$y' = -30y + \cos(t), \quad y(0) = 3, \quad \text{on} \quad t \in [0, 4].$$

What is different about the value of n you obtain for this problem? Why do you think this is happening?