HW 3 Soln

1) remember to round after each calculation

a) write x^4 as $x \cdot x \cdot x \cdot x$

$$x \cdot x = (1.01)(1.01) = 1.0201 \text{ (round)} \rightarrow 1.02$$
$$x(x \cdot x) = (1.01)(1.02) = 1.0302 \text{ (round)} \rightarrow 1.03$$
$$x \cdot (x \cdot x \cdot x) = (1.01)(1.03) = 1.0403 \text{ (round)} \rightarrow 1.04$$

$$1 - x^4 = 1 - 1.04 = -0.04$$

$$\text{exact} = 1 - (1.01)^4 = -0.04060401$$

$$\text{Error} = \frac{-0.04 - (-0.04060401)}{-0.04060401} = \boxed{-0.0148}$$

b) $(1-x^2)(1+x^2)$ from (a) $x^2 = 1.02$

$$(1-1.02)(1+1.02) = (0.02)(2.02) = -0.0404$$

no rounding needed here

$$\text{Error} = \frac{-0.0404 - (-0.04060401)}{-0.04060401} = \boxed{-0.005024}$$

c) write as $(1+x)(1-x)(1+x^2)$ Note $1+x^2$ doesnt factor

$$(1+x)(1-x)(1+x^2) = (1+1.01)(1-1.01)(1+1.02)$$
$$= (2.01)(-0.01)(2.02) \text{ (no rounding needed)}$$
$$= (-0.0201)(2.02) \text{ (no rounding needed)}$$
$$= -0.040602 \xrightarrow{\text{round}} -0.0406$$

$$\text{Error} = \frac{-0.0406 - (-0.04060401)}{-0.04060401} = \boxed{-9.82 \times 10^{-5}}$$

2) Each of these problems has a subtraction of 2 nearly equal numbers. The goal is to use algebra to rewrite this subtraction and eliminate it

a) $\ln(x+1) - \ln(x) = \ln\left(\frac{x+1}{x}\right) = \ln\left(1+\frac{1}{x}\right)$

b) $\frac{1-\cos(x)}{x^2}$ use double angle formula

$$\cos(x) = 1-2\sin^2\left(\frac{x}{2}\right)$$

$$\frac{1-\cos(x)}{x^2} = \frac{1-\left(1-2\sin^2\left(\frac{x}{2}\right)\right)}{x^2} = \frac{2\sin\left(\frac{x}{2}\right)}{x^2}$$

c) This one is tricky. You need to use the difference of 2 cubes formula

$$\sqrt[3]{x+1} - 1$$

$$a^3-b^3 = (a-b)(a^2+ab+b^2)$$

Here, $a = \sqrt[3]{x+1}$ $\quad b=1$

$a^3 = x+1$ $\quad b^3=1$

$a^2 = (x+1)^{2/3}$ $\quad b^2=1$

$$\to (x+1)-1 = \left(\sqrt[3]{x+1}-1\right)\left((x+1)^{2/3}+(x+1)^{1/3}+1\right)$$

$$1 = \left(\sqrt[3]{x+1}-1\right)\left((x+1)^{2/3}+(x+1)^{1/3}+1\right)$$

$$\sqrt[3]{x+1}-1 = \frac{1}{(x+1)^{2/3}+(x+1)^{1/3}+1}$$